

## INTISARI

# KAJIAN INTEGRAL LINTASAN LÉVY DALAM MEKANIKA KUANTUM FRAKSIONAL UNTUK MEMBENTUK PERSAMAAN SCHRÖDINGER FRAKSIONAL

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Telah dilakukan kajian mengenai penerapan integral lintasan Lévy yang dibangkitkan dengan proses stokastik Lévy pada persamaan Schrödinger yang lebih umum dalam mekanika kuantum fraksional. Sebagai perbandingan, telah dikaji pula penerapan integral lintasan Feynmann dengan proses stokastik Wiener pada persamaan Schrödinger dalam mekanika kuantum standar. Terdapat dua proses stokastik yang digunakan yaitu proses stokastik Wiener dan proses stokastik Lévy. Kedua proses ini memiliki kesamaan yaitu proses yang menghasilkan fraktal. Dalam konsep fraktal, terdapat suatu nilai yang disebut sebagai dimensi fraktal. Penerapan konsep dimensi fraktal ini terletak pada persamaan difusi yang merupakan persamaan Fokker Planck pada proses stokastik Lévy dan Wiener. Pada kaji ulang ini, ditemukan dimensi fraktal Lévy sebesar  $\alpha$  dan dimensi fraktal Wiener sebesar dua. Mekanika kuantum fraksional merupakan perumuman dari mekanika kuantum standar. Suatu keadaan dalam mekanika kuantum fraksional diwakili oleh bentuk fungsi gelombang pada persamaan Schrödinger fraksional yang dibangkitkan dari kernel integral lintasan Lévy. Sedangkan keadaan dalam mekanika kuantum standar diwakili oleh bentuk fungsi gelombang pada persamaan Schrödinger standar yang dibangkitkan dari kernel integral lintasan Feynmann. Nilai kernel Lévy dan Feynmann ditinjau sedemikian rupa sehingga diperoleh bentuk yang paling sederhana dalam bentuk integral Fourier ruang momentum. Selanjutnya, dilakukan kaji ulang terhadap persamaan Schrödinger yang diteliti melalui nilai kernel integral lintasan masing-masing. Pada perhitungan akhir, diperoleh perbandingan antara nilai energi Lévy dengan energi pada mekanika kuantum standar. Perbandingan ini dibatasi pada nilai indeks  $\alpha$  pada interval  $1 < \alpha \leq 2$ . Pada saat nilai indeks  $\alpha = 2$ , diperoleh nilai energi pada mekanika kuantum standar pada kasus partikel bebas.

**Kata kunci:** Proses stokastik Lévy, integral lintasan Lévy, mekanika kuantum fraksional, persamaan Schrödinger fraksional.

## ABSTRACT

### *THE STUDY OF LÉVY PATH INTEGRAL IN FRACTIONAL QUANTUM MECHANICS FOR OBTAINING FRACTIONAL SCHRÖDINGER EQUATION*

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The implementation of Lévy path integral which is generated by Lévy stochastic process on fractional Schrödinger equation has been investigated in the framework of a fractional quantum mechanics. As the comparison, the implementation of Feynmann path integral which is generated by Wiener stochastic process on Schrödinger equation also has been investigated in the framework of a standard quantum mechanics. There are two stochastic process which have been investigated. There are Lévy stochastic process and Wiener stochastic process. Both of them have the similarity characteristic to produce fractal. In fractal's concept, there is a value which known as fractal dimension. The implementation of fractal dimension is in the diffusion equation which are obtained by using Fokker Planck equation on are Lévy stochastic process and Wiener/Brown stochastic process. In this paper, Lévy and Wiener fractal dimension have been obtained. There are  $\alpha$  for Lévy fractal dimension and 2 for Wiener/Brown fractal dimension. Fractional quantum mechanics is generalization of standard quantum mechanics. A fractional quantum mechanics state is represented by wave function from fractional Schrödinger equation. Fractional Schrödinger equation itself is generated by using kernel of Lévy path integral which is generated by Lévy stochastic process on fractional Schrödinger equation. Otherwise, a standard quantum mechanics state is represented by wave function from standard Schrödinger equation. Standard Schrödinger equation itself is generated by using kernel of Feynmann path integral which is generated by Wiener/Brown stochastic process on Standard Schrödinger equation. Both Lévy and Feynmann Kernel have been investigated and for the final output is the simplest form of those kernel in Fourier Integral momentum phase. It is found that the form of those kernel have similitaries each other. Afterthat, we obtain Schrödinger equation from Lévy and Feynmann Kernel. In the end, we find that the comparison of Lévy energy in fractional quantum mechanics and particle energy in standard quantum mechanics.

This comparison is limited in range  $1 < \alpha \leq 2$ . So, when it comes to  $\alpha = 2$ , then we obtain particle energy in standard quantum mechanics for free particle.

**Keywords:** Lévy stochastic process, Lévy path integral, fractional quantum mechanics, fractional Schrödinger equation