# INTISARI

## BEBERAPA OPERATOR PADA RUANG BERNORMA YANG DIPERUMUM

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Dalam disertasi ini, pengembangan konsep norma dalam analisis fungsional dibahas melalui konstruksi norma-norma yang diperumum, yaitu norma-$F$-2 dan norma-$G$. Masing-masing norma tersebut digunakan untuk membentuk ruang yang dinamai ruang bernorma-$F$-2 dan ruang bernorma-$G$. Berbagai sifat dasar kedua ruang tersebut, seperti kekonvergenan barisan, barisan Cauchy, dan kelengkapan ruang, diteliti dan dibuktikan. Pada ruang bernorma-$F$-2, diberikan karakterisasi kekonvergenan barisan, yang selanjutnya diperlukan dalam pembuktian kelengkapan ruang tersebut. Sementara itu, kelengkapan ruang bernorma-$G$ ditunjukkan langsung berdasarkan definisi kelengkapan.

Konsep kekontinuan dan keterbatasan operator linear dikembangkan dalam konteks ruang bernorma-$F$-2 dan ruang bernorma-$G$. Selanjutnya, diperoleh syarat cukup untuk kekontinuan operator linear pada kedua ruang tersebut, sedangkan syarat perlu masih dianggap sebagai persoalan terbuka secara umum. Namun demikian, untuk operator matriks pada ruang tertentu, syarat perlu dan cukup untuk kekontinuannya berhasil dirumuskan. Beberapa versi Teorema Banach-Steinhaus dibuktikan pada himpunan semua operator linear terbatas antara ruang-ruang bernorma yang diperumum. Untuk mendukung pembuktian tersebut, dirumuskan dan dibuktikan Teorema Kategori Baire dalam versi ruang bernorma-$G$.

Pada bagian akhir penelitian ini, didefinisikan operator superposisi pada beberapa ruang bernorma diperumum dan diteliti sifat kekontinuannya.

**Kata-kata kunci:** norma-$F$-2, norma-$G$, Teorema Banach-Steinhaus, operator matriks, operator superposisi.

## ABSTRACT

### OPERATORS ON GENERALIZED NORMED SPACES

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In this dissertation, the development of the concept of norms in functional analysis is explored through the construction of generalized norms, namely the 2-$F$-norm and the $G$-norm. These norms are used to form the 2-$F$-normed space and the $G$-normed space, respectively. Various fundamental properties of these spaces, such as sequence convergence, Cauchy sequences, and completeness, are investigated and established. In the 2-$F$-normed space, a characterization of sequence convergence is provided, which is subsequently used to prove the space's completeness. In contrast, the completeness of the $G$-normed space is shown directly from the definition.

The concepts of continuity and boundedness of linear operators are developed in the context of 2-$F$-normed space and $G$-normed space. Sufficient conditions for the continuity of linear operators in both spaces are obtained, while the necessary conditions remain generally open problems. However, for matrix operators on certain spaces, necessary and sufficient conditions for continuity are successfully formulated. Several versions of the Banach-Steinhaus Theorem are established for the set of all bounded linear operators acting between the generalized normed spaces. To support these results, a version of the Baire Category Theorem is formulated and proven within the context of $G$-normed space.

In the final part of this study, the superposition operator is defined on several generalized normed spaces and its continuity properties are examined.

**Keywords:** 2-$F$-norm, $G$-norm, Banach-Steinhaus Theorem, matrix operator, superposition operator.