# ABSTRACT

## NECESSARY AND SUFFICIENT CONDITIONS FOR $\mathcal{U}(*_k)$ TO COINCIDE WITH THE PRIME RADICAL $\beta$

By

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14/373813/SPA/502

A class $\mu$ of rings is said to be hereditary (respectively, left hereditary, right hereditary) if for every ring $R \in \mu$, we have $I \in \mu$ for every ideal (respectively, left ideal, right ideal) $I$ of $R$. A class $\mu$ of prime rings (respectively semiprime rings) is called a special (respectively weakly special) class of rings if $\mu$ contains all ideals of ring $A \in \mu$ and $\mu$ is closed under essential extensions. A prime ring $A$ is called a $*-$ring if $A$ is a prime ring and $A$ has no nonzero proper prime ideal. The essential closure $*_k$ of the class $*$ of all $*-$rings is a special class of rings. The existing of special class of rings motivated the existing of special class of modules. The weakly special class of modules will be introduced.

The existing of $*-$ring motivates the existing of $*_p-$module. We give a necessary and sufficient condition for a ring to be a $*-$ring and some further properties of special class of modules related to the problem posed by Gardner in 1988, that is, the coincidence between the prime radical $\beta$ and the upper radical $\mathcal{U}(*_k)$ generated by $*_k$. Some necessary and sufficient conditions for the question whether the prime radical $\beta$ coincide with the upper radical $\mathcal{U}(*_k)$ to have a positive answer will be given. A radical $\alpha$ is called left strong (respectively, right strong), if for every ring $R$ and every left ideal (respectively, right ideal) $L \in \alpha$ of $R$, we have $L^* \in \alpha$. It is well known that for any homomorphically closed class $\mu$ of rings, there exists the smallest strong radical $\mathcal{L}_{s(\mu)}$ containing $\mu$. A supernilpotent radical $\alpha$ is called an $N-$radical if $\alpha$ is left strong and left hereditary. Some results concerning $N-$radical are also shown and we also introduce an almost $\beta-$ring.