

REFERENCES

- Abell, M. L. and Braselton, J.P., 2019, *Introductory Differential Equations*, 5th Edition, Academic Press.
- Al-Dosary, K. I. T., 2010, Inverse integrating factor for classes of planar differential systems. *International Journal of Mathematical Analysis*, No. 29, Vol. 4, p. 1433–1446.
- Anacleto, M. and Vidal, C., 2020, Dynamics of a delayed predator-prey model with allee effect and holling type II functional response. *Mathematical Methods in the Applied Sciences*, Vol. 43, No. 9, p. 5708–5728,
- Arditi, R., Abillon, J. M. and da Silva, J. V., 1977, The effect of a time-delay in a predator-prey model. *Mathematical Biosciences*, Vol. 33, No. 1-2, p. 107–120.
- Arnold, V. I., Gusein-Zade, S. M., and Varchenko, A. N., 2012, *Singularities of Differentiable Maps*, Vol. 2, Birkhäuser Boston.
- Atay, F. M., 1998, Van der pol's oscillator under delayed feedback. *Journal of Sound and Vibration*, Vol. 218, No. 2, p. 333–339.
- Bellman, R. and Cook, K. L., 1963, *Mathematics in Science and Engineering. A series of Monographs and Textbooks. Differential-Difference Equations*, Vol. 6, 1st edition. Academic Press Inc.
- Bender, C. M. and Orszag, S. A., 2013, *Advanced mathematical methods for scientists and engineers I: Asymptotic methods and perturbation theory*. Springer Science & Business Media.
- Binatari, N., Adi-Kusumo, F. and Aryati, L., 2022, Stability regions and bifurcation analysis of a delayed predator-prey model caused from gestation period, *International Journal of Differential Equations*, Vol. 2022, p. 1–10.

- Binatari, N., Horssen, W., Verstraten, P., Adi-Kusumo, F. & Aryati, L. On the multiple time-scales perturbation method for differential-delay equations. *Nonlinear Dynamics*. **112**, 8431-8451 (2024,4)
- Bogolyubov, N. N, 2020, Perturbation theory, 2020.
- Boussaada, I. and Niculescu, S. I., 2018, On the dominance of multiple spectral values for time-delay systems with applications. *IFAC Papers OnLine*, Vol. 51, No. 14, p. 55–60, 14th IFAC Workshop on Time Delay Systems.
- Breda, D., Menegon, G. and Nonino, M., 2018, Delay equations and characteristic roots: stability and more from a single curve. *Electronic Journal of Qualitative Theory of Differential Equations*, Vol. 2018, No. 89, p. 1–22.
- Brown, J. W., Churchill, R. V., et al, 2009, *Complex variables and applications*. McGraw-Hill Higher Education, Boston.
- Burton, T. A., 1978, *Stability and Periodic Solutions of Ordinary and Functional*, Vol. 178. Dover Publications.
- Cahlon, B. and Schmidt, D., 2004, Stability criteria for certain second-order delay differential equations with mixed coefficients. *Journal of Computational and Applied Mathematics*, Vol. 170, No. 1, p. 79–102.
- Çelik C., 2008, The stability and hopf bifurcation for a predator–prey system with time delay. *Chaos, Solitons & Fractals*, Vol. 37, No. 1, p. 87–99.
- Chow, Y. K., 2001, Melnikov’s method with applications. Master’s thesis, The University of British Columbia.
- Cimen, E. and Uncu, S., 2020, On the solution of the delay differential equation via laplace transform. *Communications in Mathematics and Applications*, Vol. 11, No. 3, p. 379–387..
- Cooke, K. L. and van den Driessche, P., 1986, On zeroes of some transcendental equations. *The Funkcialaj Ekvacioj*.

- Corless., 1996, On the LambertW function. *Advances in Computational Mathematics*, Vol. 5, No. 1, p. 329-359..
- Das, S. L. and Chatterjee, A., 2002, Multiple scales without center manifold reductions for delay differential equations near hopf bifurcations. *Nonlinear Dynamics*, Vol. 30, No. 4, p. 323–335.
- Das, S. L. and Chatterjee, A., 2005, Second order multiple scales for oscillators with large delay. *Nonlinear Dynamics*, Vol. 39, No. 4, p. 375–394.
- Dence, T. P., 2013, A brief look into the lambert w function. *Applied Mathematics*, Vol. 04, No. 06, p. 887–892.
- Duffy, D. G., 2004, *Transform methods for solving partial differential equations*. Chapman and Hall/CRC, 2nd Edition.
- Erneux, T., 2005, Multiple time scale analysis of delay differential equations modeling mechanical systems. In *Volume 6: 5th International Conference on Multi-body Systems, Nonlinear Dynamics, and Control, Parts A, B, and C*. ASMEDC.
- Erneux, T., 2009, *Applied delay differential equations*, Vol. 3, Springer Science & Business Media.
- Fadai, N. T., Johnston, S. T., and Simpson, M. J., 2020, Unpacking the allee effect: determining individual-level mechanisms that drive global population dynamics. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, Vol. 476, No. 2241.
- Freedman, H. I. and Waltman, P., 1975, Perturbation of two-dimensional predator-prey equations. *SIAM Journal on Applied Mathematics*, Vol. 28, No. 1, p. 1–10.
- Ghouli, Z., Hamdi, M. and Belhaq, M., 2019, The delayed van der pol oscillator and energy harvesting. In *Springer Proceedings in Physics*, p. 89–109. Springer Singapore.

- Glass, D. S., Xiaofan Jin, X. and Riedel-Kruse, I. H., 2021, Nonlinear delay differential equations and their application to modeling biological network motifs. *Nature Communications*, Vol. 12, No. 1.
- Grossi, M., 2018, A morse lemma for degenerate critical points of solutions of nonlinear equations in \mathbb{R}^2 . *Advanced Nonlinear Studies*, Vol. 20, p. 1 – 18.
- Hale, J. K., 1966, Averaging methods for differential equations with retarded arguments and a small parameter. *Journal of Differential Equations*, Vol. 2, No. 1, p. 57–73.
- Hale, J. K., 1971, *Functional Differential Equations*. Springer US.
- Hale, J. K. and Lunel, S. M. L., 1973, *Introduction to Functional Differential Equations*. Applied Mathematical Sciences. Springer New York, 1993.
- Hamdi, M. and Belhaq, M., 2015, Quasi-periodic vibrations in a delayed van der pol oscillator with time-periodic delay amplitude. *Journal of Vibration and Control*, Vol. 24, No. 24, p. 5726–5734.
- Haque, M., Sarwardi, S., Preston, S., and Venturino, E., 2011, Effect of delay in a lotka–volterra type predator–prey model with a transmissible disease in the predator species. *Mathematical Biosciences*, Vol. 234. No. 1, p. 47–57.
- Hayes, N. D., 1950, Roots of the transcendental equation associated with a certain difference-differential equation. *Journal of the London Mathematical Society*, Vol. s1-25. No. 3, p. 226–232.
- Holmes, M. H., 2013, *Introduction to Perturbation Methods*. Springer New York.
- Hoppensteadt, F. C. and Miranker, W. L., 1977, Multitime methods for systems of difference equations. *Studies in Applied Mathematics*, Vol. 56, No. 3, p. 273–289.
- Huang, C., 2018, Multiple scales scheme for bifurcation in a delayed extended van der pol oscillator. *Physica A: Statistical Mechanics and its Applications*, Vol. 490, p. 643–652.

- Inspurger, T. and Stépán, G., 2002, Stability chart for the delayed mathieu equation. *Proceedings of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, Vol. 458, No. 2024, p. 1989–1998.
- Inspurger, T. and Stépán, G., 2003, Stability of the damped mathieu equation with time delay. *Journal of Dynamic Systems, Measurement, and Control*, Vol. 125, No. 2, p. 166–171.
- Kalmár-Nagy, T., 2009, Stability analysis of delay-differential equations by the method of steps and inverse laplace transform. *Differential Equations and Dynamical Systems*, Vol. 17, No. 1-2, p. 185–200.
- Katsikadelis, J. T., 2002, *Boundary elements: theory and applications*. Elsevier.
- Kerr, G. and González-Parra, G., 2022, Accuracy of the laplace transform method for linear neutral delay differential equations. *Mathematics and Computers in Simulation*, Vol. 197, p. 308–326.
- Kevorkian, J. and Cole, J. D., 1996, *Multiple Scale and Singular Perturbation Methods*. Springer New York.
- Kuang, Y. 1993, *Delay Differential Equations: With Applications in Population Dynamics*. Elsevier Science.
- Lakshmanan, M. and Senthilkumar, D. V., 2011, *Dynamics of Nonlinear Time-Delay Systems*. Springer Series in Synergetics. Springer Berlin Heidelberg.
- Li, D., Liu, H., Zhang, H., Ma, M., Ye, Y. and Wei, Y., 2023, Bifurcation analysis in a predator-prey model with an allee effect and a delayed mechanism. *Acta Mathematica Scientia*, Vol. 43, No. 3, p. 1415–1438.
- Liu, W. and Jiang, Y., 2018, Bifurcation of a delayed gause predator-prey model with michaelis-menten type harvesting. *Journal of Theoretical Biology*, Vol. 438, p. 116–132.
- Lotka, A. J., 1910, Contribution to the theory of periodic reactions. *The Journal of Physical Chemistry*, Vol. 14, No. 3, p. 271–274.

- Mackey, M. & Glass, L. 1977, Oscillation and Chaos in Physiological Control Systems. *Science*. **197**, 287-289.
- Michiels, W. and Niculescu, S. I., 2014, Stability, control, and computation for time-delay systems - an eigenvalue-based approach (2nd Ed.). In *Advances in design and control*.
- Milnor, J., 1963, *Morse Theory*. Princeton University Press.
- Morrison, T. M. and Rand, R. H., 2007, Resonance in the delayed nonlinear Mathieu equation. *Nonlinear Dynamics*, Vol. 50, No. 1-2, p. 341–352.
- Murdock, J., 1983, Some asymptotic estimates for higher-order averaging and a comparison with iterated averaging. *SIAM Journal of Mathematical Analysis*, Vol. 14, No. 3, p. 421–424.
- Murdock, J. and Wang, L. C., 1996, Validity of the multiple scale method for very long intervals. *ZAMP Zeitschrift für angewandte Mathematik und Physik*, Vol. 47, No. 5, p. 760–789.
- Murdock, J., 1999, *Perturbations: theory and methods*. SIAM.
- Nandakumar, K., Wahi, P. and Chatterjee, A., 2010, Infinite dimensional slow modulations in a well known delayed model for cutting tool vibrations. *Nonlinear Dynamics*, Vol. 62, No. 4, p. 705–716.
- Nayfeh, A. H., 2008, *Perturbation Methods*. Physics textbook. Wiley.
- Nayfeh, A. H., 2011, *Introduction to perturbation techniques*. John Wiley & Sons.
- Nelson, P., 2013, *Dynamical Systems Theory, Delay Differential Equations*, pages 637–641. Springer New York, New York, NY.
- Perko, L., 2001, *Differential Equations and Dynamical Systems*. Springer New York.

- Perko, L., 1969, Higher order averaging and related methods for perturbed periodic and quasi-periodic systems. *SIAM Journal on Applied Mathematics*, Vol. 17, No. 4, p. 698–724.
- Pinney, E., 1958, *Ordinary Difference-Differential Equations*. University of California Press.
- Poincare, H., 1959, *New Methods of Celestial Mechanics*, volume I and II of NASA technical translations, F-450. National Aeronautics and Space Administration.
- Pontryagin, L. S., 1959, On the zeros of some transcendental functions.
- Rafei, M. and van Horssen, W. T., 2012, Solving systems of nonlinear difference equations by the multiple scales perturbation method. *Nonlinear Dynamics*, Vol. 69, No. 4, p. 1509–1516.
- Reyn, J. W. and van Horssen, W. T., 1995, Bifurcation of limit cycles in a particular class of quadratic systems. *Differential and Integral Equations*, Vol. 8, No. 4, p. 907 – 920.
- Ruan, S. and Wei, J., 2003, On the zeros of transcendental functions with applications to stability of delay differential equations with two delays. *Dynamics of Continuous, Discrete and Impulsive Systems Series A: Mathematical Analysis*, Vol. 10, no. 6, p. 863–874.
- Sah, S. M. and Rand, R. H., 2018, Three ways of treating a linear delay differential equation. In *Springer Proceedings in Physics*, pages 251–257. Springer International Publishing.
- Saha, A., Bhattacharya, B. and Wahi, P., 2009, A comparative study on the control of friction-driven oscillations by time-delayed feedback. *Nonlinear Dynamics*, Vol. 60, No. 1-2, p. 15–37.
- Sahoo, B. and Poria, S., 2015, Effects of additional food in a delayed predator–prey model. *Mathematical Biosciences*, Vol. 261, p. 62–73.

- Sanders, J. A., Verhulst, F., and Murdock, J., 2007, *Averaging methods in nonlinear dynamical systems*, volume 59. Springer.
- Schiff, J. L., 1999, *The Laplace Transform*. Springer New York.
- Shaw, S., 2001, *PERTURBATION TECHNIQUES FOR NONLINEAR SYSTEMS*, pages 1009–1011. Elsevier.
- Sherman, M., Kerr, G. and Gonzalez-Parra, G. 2023, Analytical solutions of linear delay-differential equations with dirac delta function inputs using the laplace transform. *Computational and Applied Mathematics*, Vol. 42, No. 6.
- Smith, H., 2010, *An Introduction to Delay Differential Equations with Applications to the Life Sciences*. Springer.
- Subramanian, R. and Krishnan, A., 1979, Non-linear discrete time systems analysis by multiple time perturbation techniques, *Journal of Sound and Vibration*, Vol. 63, No. 3, p. 325–335.
- van Horssen, W. T and ter Brake, M., 2009, On the multiple scales perturbation method for difference equations, *Nonlinear Dynamics*, Vol. 55, No. 4, p. 401–418.
- van horssen, W. T. and Kooij, R. E., 1994, Bifurcation of limit cycles in a particular class of quadratic systems with two centers, *Journal of Differential Equations*, Vol. 114, No. 2, p. 538–569.
- Verhulst, F., 1996, *Nonlinear Differential Equations and Dynamical Systems*, Springer Berlin Heidelberg.
- Verhulst, F., 2005, *Methods and Applications of Singular Perturbations*, Springer New York.
- Verstraten, P., 2019, A perturbation method for delay differential equations, thesis, TU Delft.

- Wahi, P. and Chatterjee, A., 2005, Regenerative tool chatter near a codimension 2 hopf point using multiple scales. *Nonlinear Dynamics*, Vol. 40, No. 4, p. 323–338.
- Wang, H. and Hu, H., 2003, Remarks on the perturbation methods in solving the second-order delay differential equations. *Nonlinear Dynamics*, Vol. 33, No. 4, p. 379–398.
- Wirkus, S. and Rand, R., 2002, The dynamics of two coupled van der pol oscillators with delay coupling. *Nonlinear Dynamics*, Vol. 30, No. 3, p. 205–221.
- Woldaregay, M. M., 2022, Solving singularly perturbed delay differential equations via fitted mesh and exact difference method, *Research in Mathematics*, Vol. 9, No. 1.
- Wright, E. M., 1946, The non-linear difference-differential equation. *The Quarterly Journal of Mathematics*, Vol. 17, No. 1, p. 245–252.
- Yan, X. P. and Chu, Y. D., 2006, Stability and bifurcation analysis for a delayed lotka–volterra predator–prey system. *Journal of Computational and Applied Mathematics*, Vol. 196, No. 1, p. 198–210.
- Ye, Y., Liu, H., Wei, Y., Zhang, K., Ma, M. and Ye, J., 2019, Dynamic study of a predator-prey model with allee effect and holling type-i functional response. *Advances in Difference Equations*, Vol. 2019, No. 1.
- Zelei, A., Milton, J., Stepan, G. and Insperger, T., 2021, Response to perturbation during quiet standing resembles delayed state feedback optimized for performance and robustness. *Scientific Reports*, Vol. 11, No. 1.
- Zhang, J. F. and Huang, F., 2014, Nonlinear dynamics of a delayed leslie predator–prey model. *Nonlinear Dynamics*, Vol. 77, No. 4, p. 1577–1588.
- Zoladek, H., Melnikov functions in quadratic perturbations of generalized lotka–volterra systems. *Journal of Dynamical and Control Systems*, Vol. 21, No. 4, p. 573–603.

Zwillinger, D., 1997, Handbook of differential equations, 3rd edition. Academic Press.