INTISARI

Diberikan ring topologis R dan R-modul topologis M, modul U disebut modul M-injektif topologis jika untuk setiap monomorfisma kontinu $f\colon K \to M$, dengan K submodul terbuka M, dan setiap homomorfisma kontinu $g\colon K \to U$, terdapat homomorfisma kontinu $h\colon M \to U$ sedemikian hingga $hf = g$. Modul M-injektif topologis merupakan modul injektif topologis relatif terhadap seluruh R-modul topologis di kategori $Top_{\sigma[M]}$. Amplop M-injektif topologis dari modul topologis N merupakan modul M-injektif minimal yang memuat N. Keinjektifan jumlahan langsung tak berhingga modul-modul injektif tidak selalu dapat berlaku. Selain itu, eksistensi amplop injektif belum tentu ada untuk sebarang kategori. Sifat kompak pada ruang topologis erat kaitannya dengan sifat keberhinggaan di dalam ruang topologi. Untuk setiap modul topologis di $Top_{\sigma[M]}$, tidak selalu terdapat modul M-injektif topologis kompak yang memuatnya. Dalam disertasi ini dikaji keinjektifan jumlahan langsung modul-modul injektif topologis di $Top_{\sigma[M]}$, eksistensi amplop M-injektif topologis, dan dianalisis kekompakan modul topologis di $Top_{\sigma[M]}$. Jumlahan langsung tak berhingga modul-modul M-injektif topologis merupakan modul M-injektif topologis jika jumlahan langsung merupakan submodul terbuka dari hasil kali langsung modul-modul M-injektif topologis. Amplop M-injektif topologis dari modul topologis (N, τ_N) merupakan *trace* topologis M di E_{τ_N} dengan E_{τ_N} amplop injektif topologis N di $Top_{R\text{-}MOD}$. Pada modul topologis nonprekompak dan modul topologis kompak secara lokal, tidak terdapat modul M-injektif topologis kompak yang memuatnya, sedangkan pada modul topologis kompak dapat termuat pada suatu modul M-injektif topologis kompak.

Kata-kata kunci: Modul M-Injektif Topologis, Jumlahan Langsung Modul M-Injektif Topologis, Amplop M-Injektif Topologis, Kekompakan Modul Topologis

Abstract

Let R be a topological ring and M be a topological R-module. A module U is called a topological M-injective module if for every continuous monomorphism $f\colon K \to M$, where K is an open submodule of M, and every continuous homomorphism $g\colon K \to U$, there exists a continuous homomorphism $h\colon M \to U$ such that $hf = g$. A topological M-injective module is injective relative to all topological R-modules in the category $Top_{\sigma[M]}$. The topological M-injective hull of a topological module N is the minimal M-injective module containing N. The injectivity of an infinite direct sum of injective modules does not always hold. Moreover, the existence of injective hulls is not guaranteed in every category. The compactness property in topological spaces is closely related to the finiteness property in topology. For every topological module in $Top_{\sigma[M]}$, there does not always exist a compact M-injective topological module containing it. This dissertation investigates the injectivity of direct sums of topological injective modules in $Top_{\sigma[M]}$, the existence of topological M-injective hulls, and analyzes the compactness of topological modules in $Top_{\sigma[M]}$. An infinite direct sum of topological M-injective modules is a topological M-injective module if the direct sum is an open submodule of the direct product of the topological M-injective modules. The topological M-injective hull of a topological module (N, τ_N) is the topological trace of M in E_{τ_N}, where E_{τ_N} is the topological injective hull of N in $Top_{R\text{-}MOD}$. For non-precompact topological modules and locally compact topological modules, there is no compact topological M-injective module containing them, whereas compact topological modules can be contained in a compact topological M-injective module.
Keywords: Topological M-Injective Modules, Direct Sum of Topological M-Injective Modules, Topological M-Injective Hulls, Compactness of Topological Modules